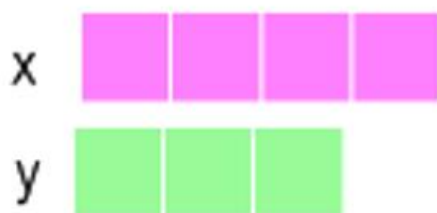


# Bar Modelling Handbook Primary to GCSE

$$x : y = 4 : 3$$



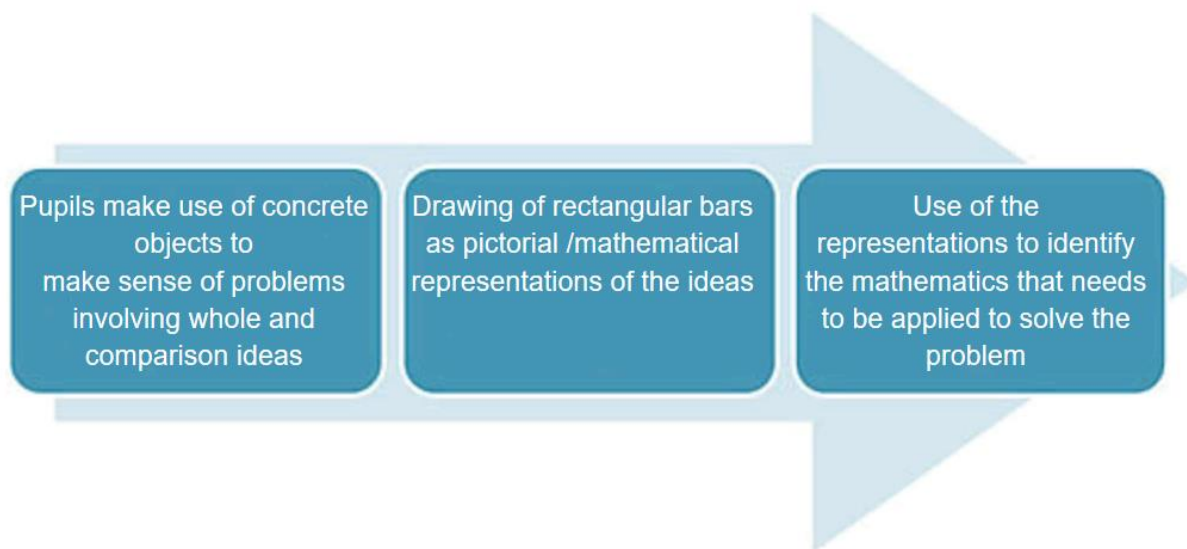
Write equations in terms of  $y$  and  $x$

## Bar Modelling

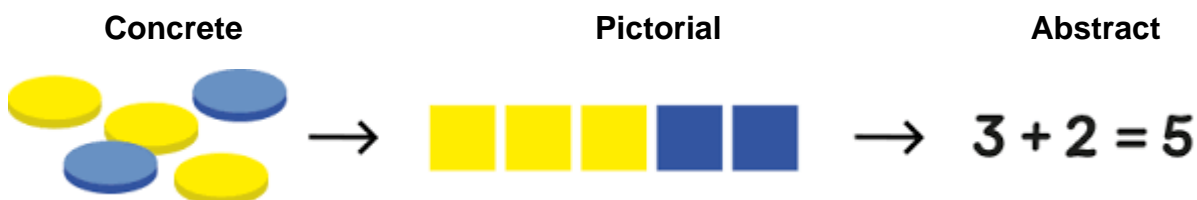
The bar can be a valuable representation to enable students to represent problems in such a way that the mathematical structure is exposed.

This enables students to 'see' the problem clearly and to then recognise the strategy they need to solve the problem.

NCETM



**Avoid the common mistakes!** Do not jump straight into the pictorial representation.

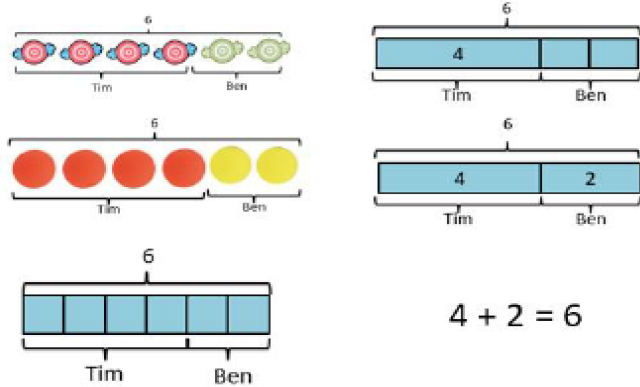


It is important to go through all the stages from concrete to pictorial to abstract at the start so that students can make sense of the problem and build up from something concrete to an abstract method that they can use fluently.

## ALL STEPS ARE IMPORTANT



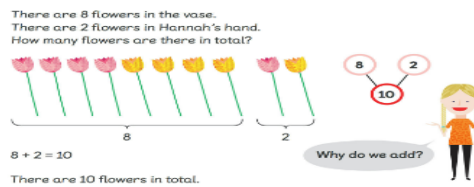
## Primary example from White Rose



## Maths No Problem – One of the recommended Primary Mastery Textbooks

<https://mathsnoproblem.com/en/mastery/bar-modelling/>

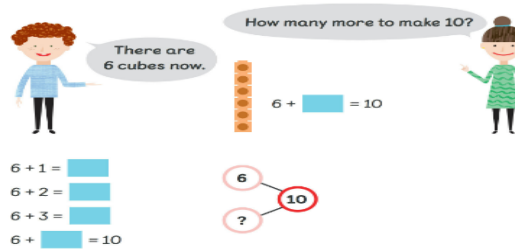
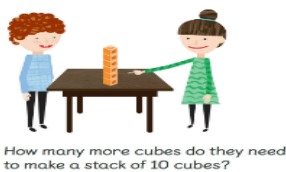
### Concrete - modelling with real objects



Reference: Maths – No Problem! Primary Maths Series Textbook 18, page 22

**Concrete – real life objects**

### Concrete - handling real objects

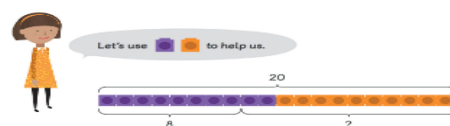


Reference: Maths – No Problem! Primary Maths Series Textbook 18, page 24

**Concrete – handling real objects (manipulatives)**

**blocks, cuisenaire rods, counters etc**

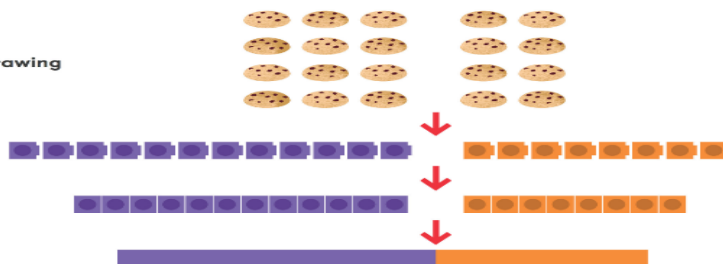
### Concrete - modelling with other objects and pictures



Reference: Maths – No Problem! Primary Maths Series Textbook 18, page 26

**Concrete – bar model**

### Concrete to pictorial - drawing



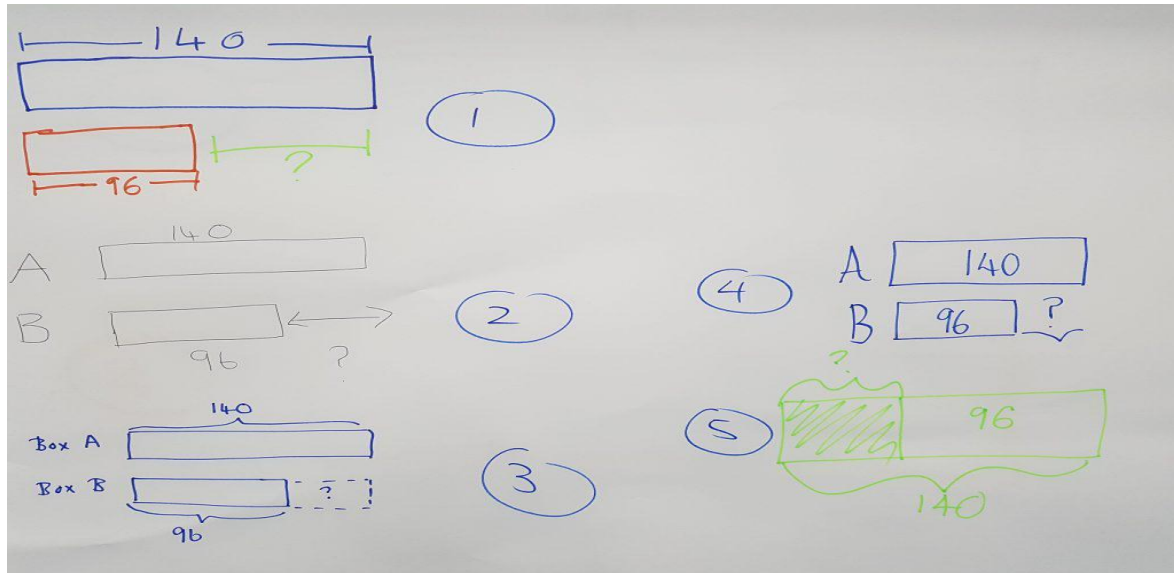
**Real – manipulative – bar model**

In later KS3 and 4 examples we will look at developing fluency by moving from bar model to abstract- calculation and abstract – algebra

### Consistency

<https://www.teachwire.net/news/8-mistakes-schools-make-teaching-the-bar-model-method-in-maths>

Teachers from over 10 schools were asked to model  $140 - 96$ . A variety of responses were given.



A consistent approach will always result in a more efficient use of bar models throughout the school and lead to fewer pupils being confused and fewer mistakes from both teachers and pupils.

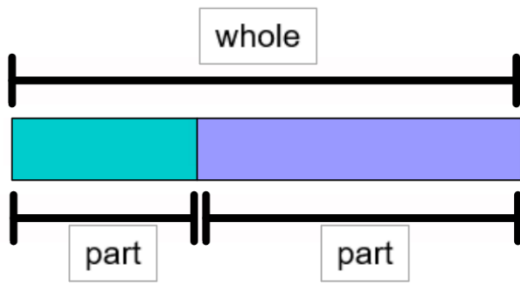
**What differences can you see?**

**What difference does it make?**

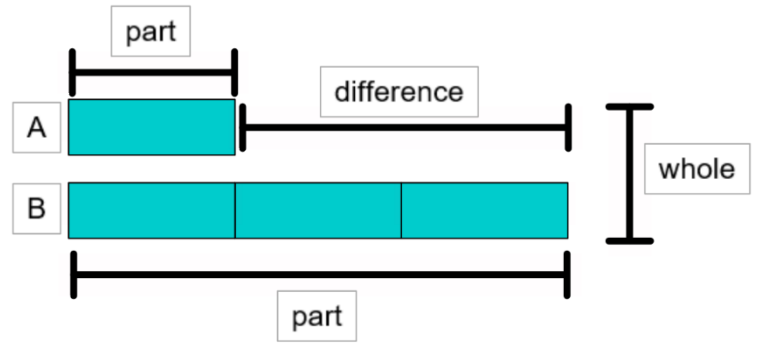
**Which model would you go for and why?**

## Choosing the correct model

### Part Whole Model



### Comparison Model

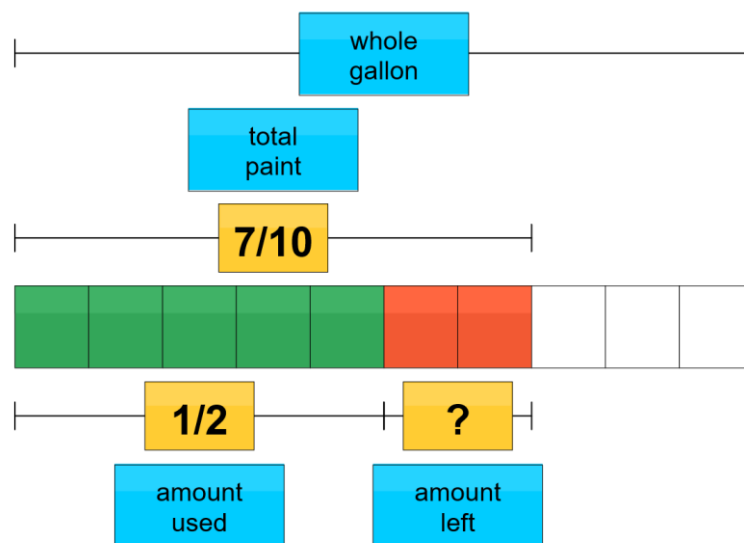


Can you think of three different questions to go with each model?

## Part Whole Model

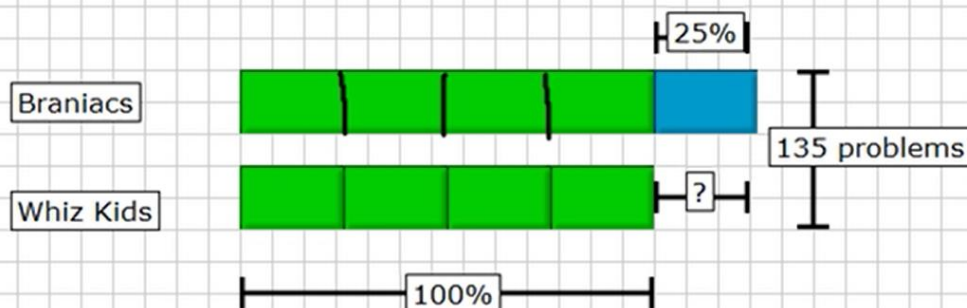
[https://www.mathplayground.com/tb\\_fractions/index.html](https://www.mathplayground.com/tb_fractions/index.html)

Kayla bought  $\frac{7}{10}$  of a gallon of blue paint. She used  $\frac{1}{2}$  a gallon to paint the shed. How much blue paint did she have left?



## Comparison Model

The Braniacs and the Whiz Kids have solved a total of 135 math problems. The Braniacs solved 25% more problems than the Whiz Kids. How many problems do the Whiz Kids have to solve to tie the score?



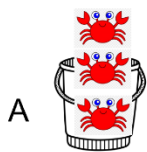
$$9 \text{ blocks} = 135$$
$$1 \text{ block} = 135 \div 9 = 15 \text{ math problems}$$

[https://www.mathplayground.com/ThinkingBlocks/thinking\\_blocks\\_decimals\\_percent\\_5.html](https://www.mathplayground.com/ThinkingBlocks/thinking_blocks_decimals_percent_5.html)

Thinking Blocks on mathplayground has a wide variety of interactive problems for students to try along with videos and a modelling tool.

## Ratio Question(s) – Introducing the bar model

Alfie and Billy go crabbing. Each bucket holds the same number of crabs. Alfie has one bucket and Billy has 3 buckets. If Alfie has 3 crabs, how many crabs does Billy have?



*Concrete - real*

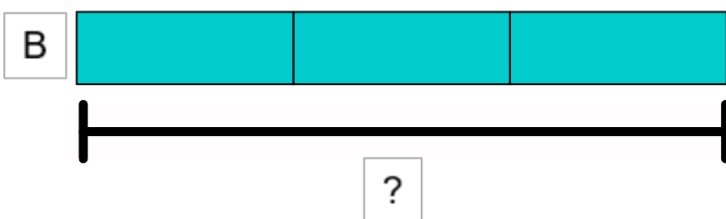
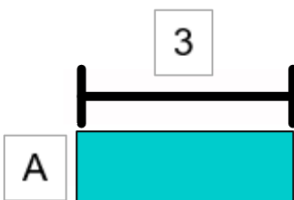
Step 1: Get students to use plastic cups and counters to model this.

Step 2: Model using counters



*Concrete - manipulatives*

Step 3: Model using bars



*Pictorial – bar model*

*Abstract - calculation*

*Abstract - algebra*

1 block = 3

$b = 3$

3 blocks =  $3 \times 3 = 9$

$3b = 9$

Ans: Billy has 9 crabs

Try these questions using CPA.

Billy has 15 crabs. How many crabs does Alfie have?

There are 28 crabs in total. How many crabs does Billy get?

Billy has 20 more crabs than Alfie. How many crabs are there in total?

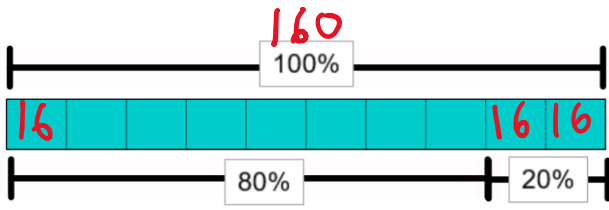
When students are familiar with bar models you can start with the bar model and move on to develop fluency with abstract calculations and algebra.

## Percentage Question(s) - From bar model to box model

(Van Hiele, proportion matrix, 'Structure and Insight' (1986), chapter 28) and

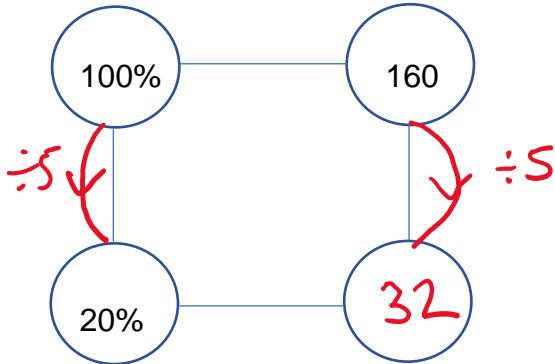
[https://drive.google.com/file/d/19OsyPhuoKM\\_16rEV1cZmOHKLpNUG5WnX/view](https://drive.google.com/file/d/19OsyPhuoKM_16rEV1cZmOHKLpNUG5WnX/view) - Don Steward - Median)

What is 20% of 160?



$$20\% \text{ of } 160 = 32$$

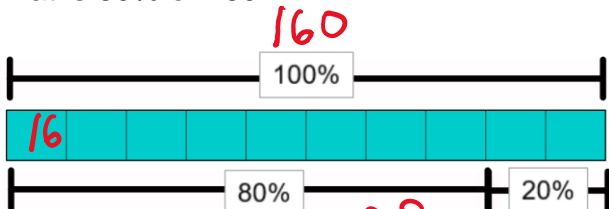
$$16 \times 2 = \underline{\underline{32}}$$



$$20\% \text{ of } 160 = 32$$

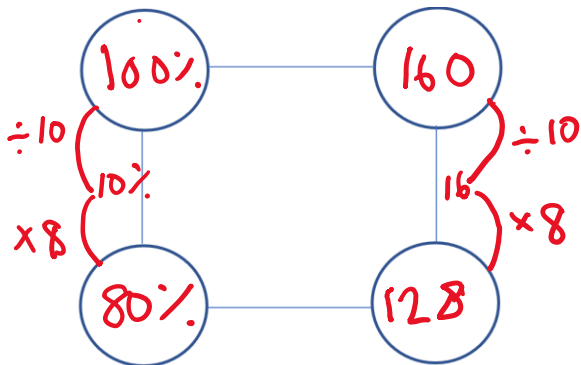
Start with single step problems and then move onto two step problems.

What is 80% of 160?



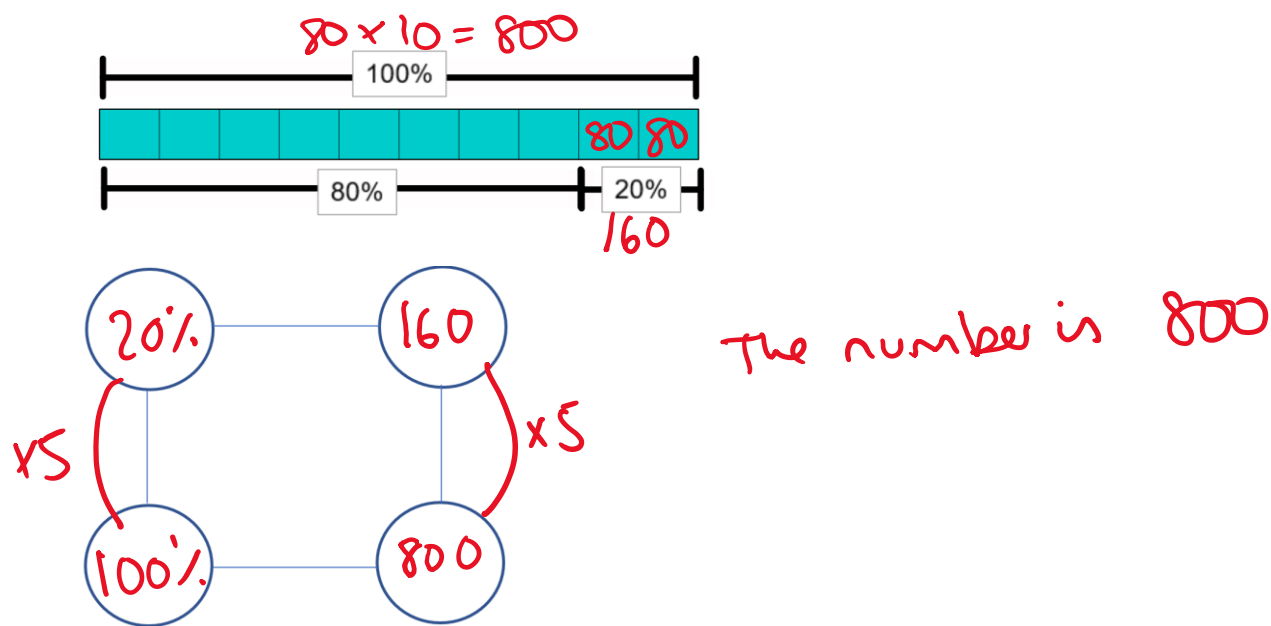
$$16 \times 8 = 128$$

$$80\% \text{ of } 160 = 128$$



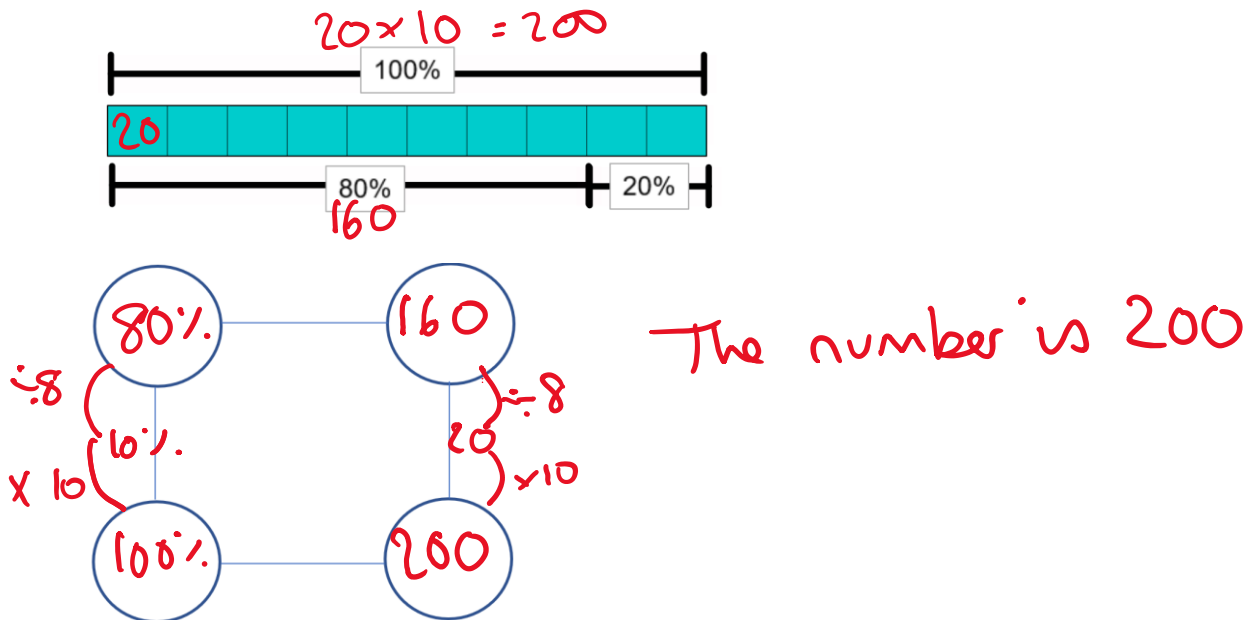


20% of a number is 160. What is the number?



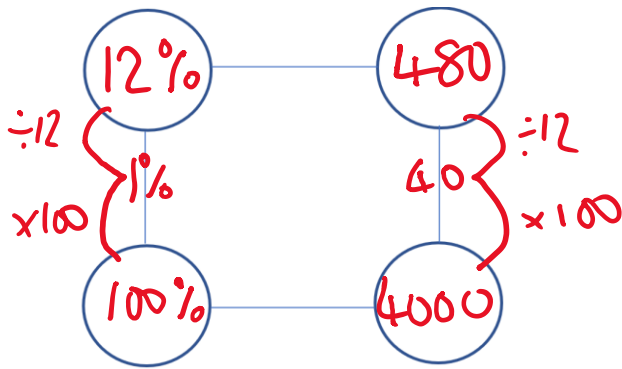
Start with single step problems and then move onto two step problems.

80% of a number is 160. What is the number?



From here you can introduce the unitary method:

12% of a number is 480. What is the number?



The number is 4000

**SSDD Questions** <https://ssddproblems.com/>

Increase £32 by 20%

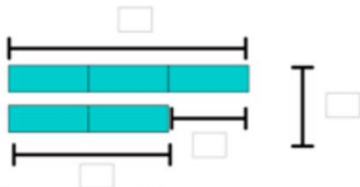
The sale price of a t-shirt is £32.00 in a 20% off sale. Work out the normal price.

The normal price of a t-shirt is £32. Today there is a 20% off sale. Work out the sale price.

£32 is 20% of x. Work out x

A good same surface, different depth question uses the same numbers but different questions. This highlights the difference in the questions. This is a great example of variation.

Alfie and Billy share 15 sweets in the ratio 3:2. How many sweets does Billy get?

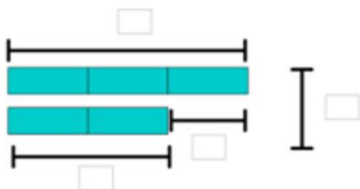


Alfie and Billy share 15 sweets in the ratio 3:2. How many sweets does Alfie get?

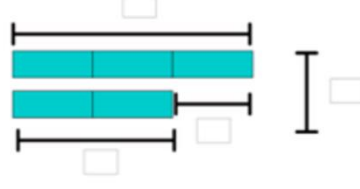


**What's the same, what's different and why?**

Alfie gets 15 sweets. How many do they get altogether?



Alfie gets 15 more sweets than Billy. How many sweets does Billy get?



Try the following NCETM (adapted) problems using bar models.

Choose the correct model - Annotate the model – Calculation - Algebra

### Year 6 Problems

1. Three quarters of a number is 54. What is the number?
2. There are 36 packets of biscuits. One half are chocolate, a ninth are digestive and a third are wafer biscuits. The rest are ginger nuts. How many biscuits are ginger nuts?
3. There is 20% off in a sale. How much would a track suit cost, if the normal price was £44.50?
4. There is 20% off in a sale. The reduced price of the jeans is £36. What was the original price?
5. At a dance there are 4 girls to every 3 boys. There are 63 children altogether? How many girls are there?
6. Seven in every nine packets of crisps in a box are salt and vinegar. The rest are plain. There are 63 packets of salt and vinegar crisps. How many packets of plain crisps are there?

### Key Stage 3 Problems

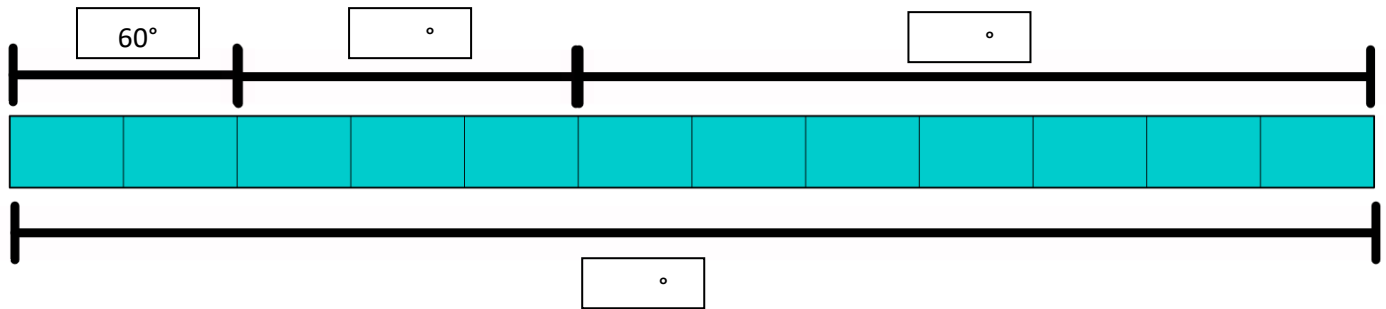
1. Ralph posts 40 letters, some of which are first class, and some of which are second class. He posts four times as many second class letters as first. How many of each class of letter does he post? (This question appeared on a GCSE higher tier paper.)
2. A computer game was reduced in a sale by 20% and it now costs £55 What was the original cost?
3. Sally had a bag of marbles. She gave one-third of them to Rebecca, and then one quarter of the **remaining** marbles to John. Sally then had 24 marbles left in the bag. How many marbles were in the bag to start with?
4. Sam bakes a variety of biscuits.  
13 are peanut, 12 are raisin, the remaining 5 were oat. If you choose 1 biscuit at random, what is the probability that you will get an oat biscuit?
5. Tom spent 30% of his pocket money and put away 45% into his savings. He was left with £2.50. How much pocket money did he receive?
6. Two numbers are in the ratio 4:5. They both sum to 135. Identify both numbers.
7. Two numbers are in the ratio 5 : 7. The difference between the numbers is 12. Work out the two numbers.
8. A herbal skin treatment uses yoghurt and honey in the ratio 5 : 3. How much yoghurt is needed to mix with 130 g of yoghurt?

## AQA GCSE Exam Questions

### Part whole

Three angles are in the ratio 2 : 3 : 7  
The smallest angle is  $60^\circ$ .

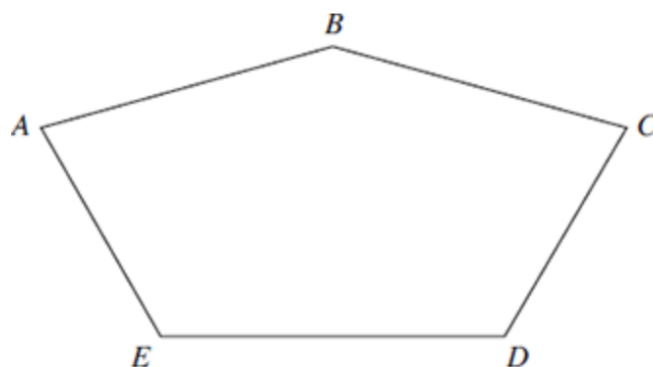
Show that these three angles will fit together at a point with no gaps.



This pentagon has a **vertical** line of symmetry.

The ratio of angles  $B : C : D = 6 : 3 : 4$

Not drawn accurately

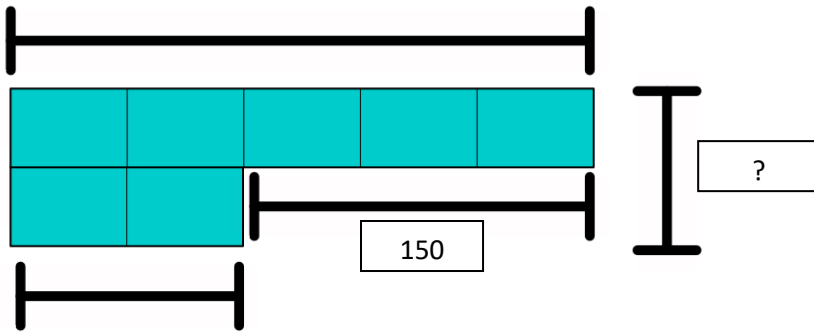


Work out the size of angle  $B$ .

## Comparison

Jon and Nik share some money in the ratio 5 : 2  
Jon gets £150 more than Nik.

How much money do they share altogether?



The sum of two numbers is 35  
The second number is four times the first number.

Work out the two numbers.

## Comparison and part whole

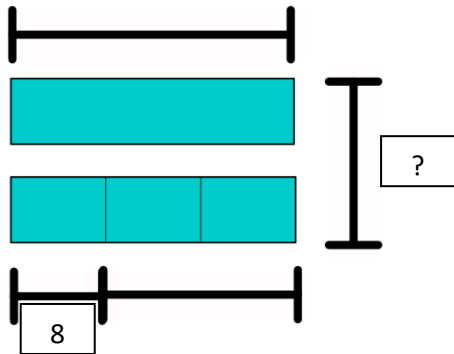
In a school show,

girls : boys = 1 : 1

girls who sing : girls who do not sing = 1 : 2

8 girls **sing** in the show.

How many students are in the show altogether?



Ali, Beth and Clare take a test.

The ratio of Ali's score to Beth's score is 5 : 3  
Ali scored 10 more marks than Beth.

Clare scored 7 more marks than Ali.

Work out each of their scores.

A toy box contains red, yellow, blue and green bricks.

25% of the bricks are red.

There are 12 yellow bricks.

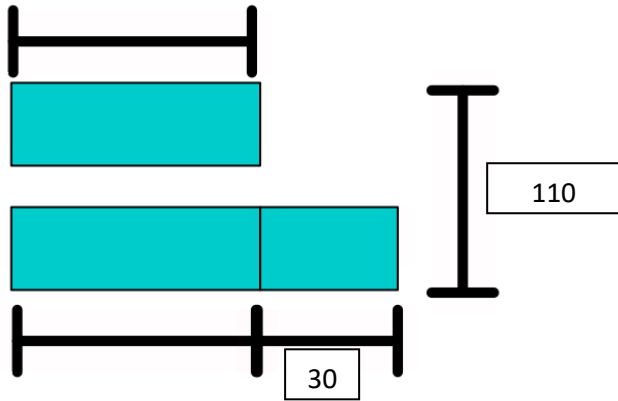
The ratio yellow to blue to green is 2 : 3 : 1

Show that there are 48 bricks in the box

## Comparison and fraction/percentages

Tom has £30 more than Ann.  
They have £110 in total.

What fraction of the total does Tom have?



There are 40% more black balls than white balls in a bag.

Work out the ratio of black balls to white balls.  
Give your answer in its simplest form.



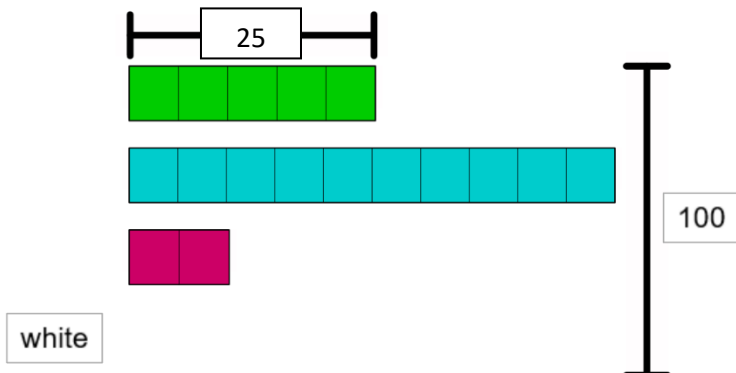
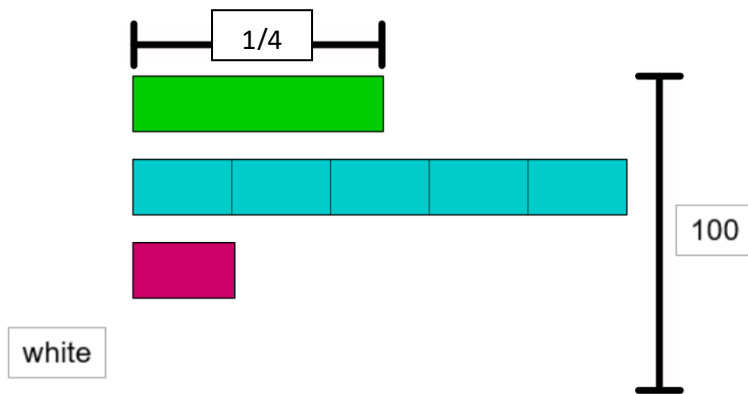
## Comparison and equivalent fractions

There are 100 balls in a bag.  
The balls are red, blue, green or white.

The ratio of blue to red is 5 : 1  
There are twice as many blue as green.

$\frac{1}{4}$  of the balls are green.

How many white balls are in the bag?



## Comparisons with changes

The ratio of the number of boys to girls at a party is 3 : 4

Six boys leave the party.

The ratio of the number of boys to girls at the party is now 5 : 8

Work out the number of girls at the party.

### Before change



### After change



Number of girls remains the same. To compare like for like change 3:4 to 6:8



Sarah has £135  
Gemma has £70  
Beth has £35

Sarah gives some money to Gemma and Beth.

The ratio of the amount of money Sarah, Gemma and Beth have **now** is 3 : 2 : 1

How much money did Sarah give to Gemma?

There are some boys and girls at a bus stop.

11 girls get on the first bus to arrive.

The number of boys and girls at the bus stop is now the same.

16 boys get on the second bus to arrive.

The ratio of the number of boys to the number of girls at the bus stop is now 1 : 3

How many girls were at the bus stop to start with?

## Compare a:b to b:c

$a$ ,  $b$  and  $c$  are positive integers.

$$a : b = 5 : 6 \quad \text{and} \quad b : c = 8 : 11$$

Work out the **smallest** possible value of  $a + b + c$



To compare like for like  $b$  must be the same length (LCM).

$$5 : 6 \quad \text{and} \quad 8 : 11$$

$$20 : 24 \quad 24 : 33$$

$$a + b + c = 20 + 24 + 33 = 77$$

At a concert the ratio of men to women is  $5 : 3$

The ratio of women to children is  $7 : 4$

Show that more than half of the people at the concert are men.

$$m : w$$

$$5 : 3$$

$$35 : 21$$

$$w : c$$

$$7 : 4$$

$$21 : 12$$

$$m : w : c = 35 : 21 : 12$$

35 are men

51.5% are men

68

During a game, players can win and lose counters.

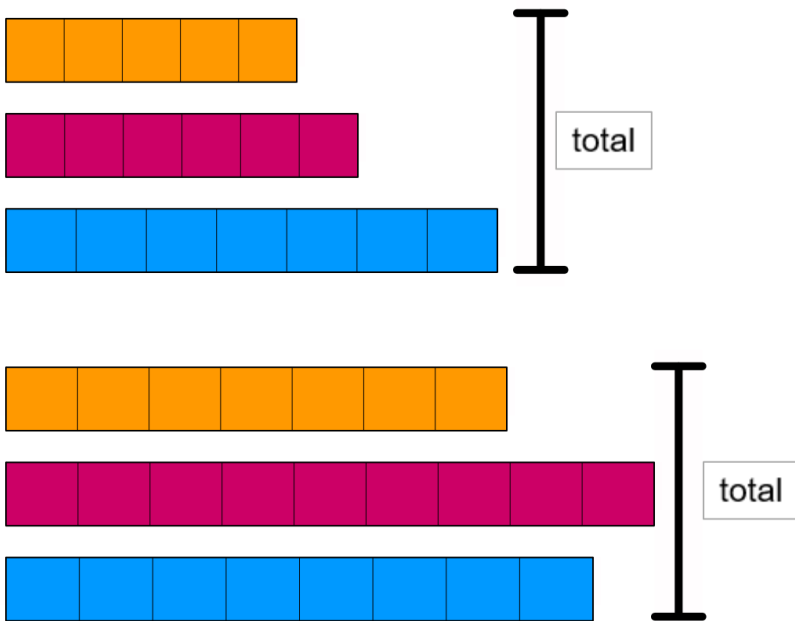
At the start of the game

Rob, Tim and Zak share the counters in the ratio 5 : 6 : 7

At the end of the game

Rob, Tim and Zak share the **same number** of counters in the ratio 7 : 9 : 8

Show that Rob ends the game with more counters than he started with.



**5 : 6 : 7 18 blocks**

**7 : 9 : 8 24 blocks**

**The total number of blocks needs to be the same. LCM of 18 and 24 is 72**

**20 : 24 : 28 72 blocks**

**21 : 27 : 24 72 blocks**

**Rob ends with one more block**

## 1:n and n:1

Which ratio is **not** equivalent to the ratio 3 : 4  
Circle your answer.

$$\frac{3}{4} : 1$$

$$1 : \frac{4}{3}$$

$$1 : 1.3$$

$$6 : 8$$



Should be 1.33333333333...

$$\frac{1}{2} : \frac{2}{3} = x : 1$$

Circle the value of  $x$ .

$$\frac{1}{3}$$

$$\frac{3}{5}$$

$$\frac{3}{4}$$

$$\frac{4}{3}$$

$y$  is 100% **more** than  $x$ .

Circle the ratio  $x : y$

$$1 : 100$$

$$100 : 1$$

$$1 : 2$$

$$2 : 1$$

**$x:y = a:b$  to fraction**

$a : b = 4 : 3$

Circle the correct statement.

$b$  is  $\frac{4}{7}$  of  $a$

$b$  is  $\frac{3}{7}$  of  $a$

$b$  is  $\frac{4}{3}$  of  $a$

$b$  is  $\frac{3}{4}$  of  $a$



$x : y : z = 2 : 3 : 5$

Circle the value of  $x$  as a fraction of  $x + y + z$

$\frac{1}{5}$

$\frac{1}{4}$

$\frac{2}{3}$

$\frac{2}{5}$

In a class, the ratio boys : girls is  $x : y$

Circle the fraction of the class that are girls.

$\frac{x}{y}$

$\frac{y}{x}$

$\frac{x}{x+y}$

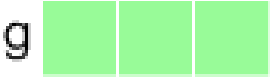
$\frac{y}{x+y}$

**x:y = a:b to equation**

Write as many equations as you can connecting r and g.



$r = \frac{2}{3}g$



$g = 1.5r$     $g = \frac{3}{2}r$

$r:g = 2:3$

$3r = 2g$

$a : b = 9 : 4$    and    $10b = 7c$

Work out  $a : c$  in its simplest form.

If  $10b = 7c$ ,  $b:c = 7:10$

$a : b$

$9 : 4$

$63 : 28$

$b : c$

$7 : 10$

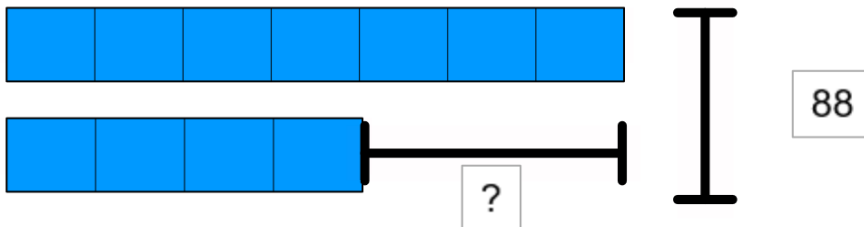
$28 : 40$

$a : c = 63 : 40$

$x : y = 7 : 4$

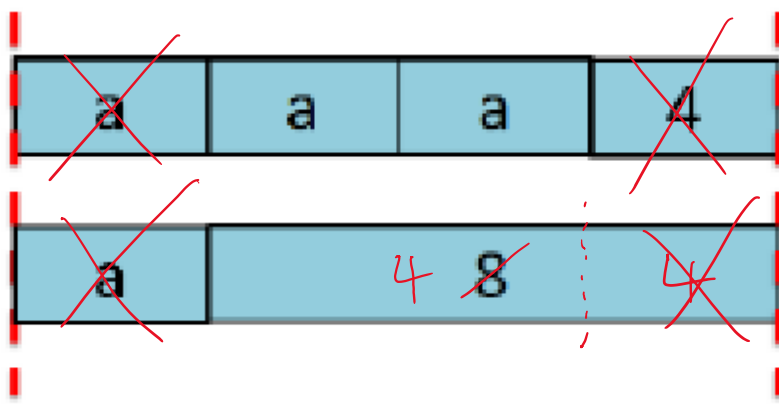
$x + y = 88$

Work out the value of  $x - y$





Is a bar model always the most appropriate model?

| $3a + 4 = a + 8$                                                                                                                  |                                                                                                                                                                     |
|-----------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p style="text-align: center;"><u>Model</u></p>  | <p style="text-align: center;"><u>Calculations</u></p> $\begin{array}{r} 3a + 4 = a + 8 \\ -a \quad -a \\ 2a + 4 = 8 \\ -4 \quad -4 \\ 2a = 4 \\ a = 2 \end{array}$ |

Now draw a model for  $3a - 4 = 8 - a$

| $3a - 4 = 8 - a$                                |                                                        |
|-------------------------------------------------|--------------------------------------------------------|
| <p style="text-align: center;"><u>Model</u></p> | <p style="text-align: center;"><u>Calculations</u></p> |

More resources and the latest professional development details available on <https://www.enigmamathshub.co.uk/>



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