Day 1

Today we were invited to Shanghai Normal University which trains 70% of the maths teachers in the Shanghai region. We were lectured by Professor Gu, who is an expert on the use of variation in teaching.

Instead of focusing on variation, however, he showed us the four main principles that underpinned what changed in the curriculum design for Shanghai practice to develop to where it is now:
1. Analyse the content to be delivered - key competencies
2. Carefully consider the order - curriculum order
3. Highlight the key point - what is at the heart of the structure in the maths that needs to be delivered
4. Set up the learning steps - provide students the opportunity to experience the structure

I was struck by the relevance of these to UK Secondary Schools.

Day 2

Day 2 in China and I spent my first day in school. I visited a middle school which taught grade 6-9 or year 7 to 10. I observed the equivalent of year 8 classes in 3 lessons.

The lessons were constructed with extreme precision and examples were chosen and sequenced expressly to expose mathematical structure.

For example, in one lesson which was titled efficient methods to multiply two digit numbers, it began with four questions:
15 x 15, 45 x 45, 65 x 65, 75 x 75

Students used different methods and then explored the solutions:
15 x 15 = 225
45 x 45 = 2025
65 x 65 = 4225
75 x 75 = 5625

They noticed that 5 x 5 = 25 and that if you multiplied the tens digit by 1 more you got the start of the answer (1x2=2, 4x5=20 etc). The teacher illustrated that the fact that all of the examples ended in 5 was significant by showing that 13 x 13 = 169 not 209.

Then they looked to generalise - this is normal and the students were confident and fluent moving between algebra and number - they were not distinct from each other to the students. They arrived at (10a + 5)(10a + 5) then 100a^2 + 100a + 25 then 100a(a + 1) + 25.

This took about 15 mins and the next questions were given:

14 x 16
43 x 47
62 x 68
71 x 79

Which the students quickly worked out to be:

14 x 16 = 224 (1x2 and 4x6)
43 x 47 = 2021 (4x5 and 3x7)
62 x 68 = 4216 (6x7 and 2x8)
71 x 79 = 5609 (7x8 and 1x9)

The significance of the unit digits adding to 10 and the appreciation of the requirement for the zero in the 4th one was stressed before further generalisation:

((10a + 5) + b) ((10a + 5) - b)
(10a + 5)^2 - b^2
Or, linking to previous
100a(a + 1) + 25 - b^2
Ths previous subtract b squared. Which then connected/led to the difference of two squares.

Not only a very interesting approach to explore the difference of two squares, but more generally, it was so impressive to see that the students had fluency in number which led to them being able to confidently work with algebra and their fluency with algebra helped them to see the structure of the number work. It was clear that we need to improve our student's familiarity and confidence with algebra but integrating it more into our number work.

Day 3: Revision Lessons in China

As you might imagine, after day 1 in schools, I was mesmorised by the complexity and standard of maths in the Chinese classroom, that my attention was mainly on that. Having had time to reflect on the maths we saw, it might be more useful to pass on ideas and concepts that are more generally applicable to our development of practice instead of an individual lesson idea.

With that in mind, I observed a lesson that was quite similar to our practice in the UK but different in one specific way. It was a revision lesson. In the lesson, student mistakes were displayed (photos of student work) and students had to find the mistake and correct the mistake. We do this in our classrooms and often use examples of student work to illustrate common errors and misconceptions. There was, however, one main difference. This sort of lesson was standard, it was not a one off, or a revision lesson for an assessment, it happened towards the end of every unit of work in the scheme. So, at the end of every topic of work, there would be this lesson where the students would be shown and correct the mistakes of the class (and it was mistakes from this specific class, that had been collected over the course of all the lessons for that topic).

I thought this was extremely effective and powerful, because not only was it highlighting and correcting misconceptions, it was bespoke - it highlighted and corrected the mistakes of that specific class. These may have been corrected in the lesson or homework that they occurred in, but that might have only benefitted the student that was corrected or at best the students nearby. This lesson allowed every student to see what misconceptions can occur and thus what not to do (because after an amount of time, they might have made a similar mistake). This, to me, seems like a very simple lesson to emulate, so please do give this a go - I know I will be trying this when I return.

Day 4: Primary school

I went back to primary school today and after the maths I had been trying to keep up with in middle school (year 7 to 10 in China), I felt like I needed it! This was a fantastic experience and I saw two very different but lovely lessons.

The first was on averages and applications - it was the final lesson in a sequence on averages - where students were presented with a box of oranges and asked how to find the mass. They came up with different plans and then were allowed to use scales to weight a number of oranges and find the average and then work out the mass (they were eventually told how many oranges there were in the box).
The focus was entirely on the thinking - How could we do this? Which methods were efficient? Which methods were accurate? I am sure many of you have done similar lessons in the UK, but I think the clear focus on formulation of a plan, refinement and then solving the problem was likely more evident here in China, as we might focus more on the calculation - here that was secondary.

The second lesson was on the laws of Commutativity, Associativity and Distributativity. For those of you that are unaware or have forgotten these terms, you do know them:
Commutativity: (addition) $a + b = b + a$ (multiplication) $a \times b = b \times a$
Associativity: (addition) $(a + b) + c = a + (b + c)$ (multiplication) $(a \times b) \times c = a \times (b \times c)$
These two simply tell us that order does not matter for + or x but it does for - or /
Distributivity: $a(b + c) = ab + ac$ [Can be written $a \times (b + c) = a \times b + a \times c$]
This is expanding brackets.
They explore these through calculation and ask students to try and find efficient methods.

Take 25 x 28, for example - students came up with different methods:

1. $25 \times 4 \times 7 = 100 \times 7 = 700$
2. $25 \times (20 + 8) = 500 + 200 = 700$
3. $(20 + 5) \times 28 = 560 + 140 = 700$
4. $5 \times 5 \times 28 = 5 \times 140 = 700$
5. $25 \times (30 - 2) = 750 - 50 = 700$
6. $5 \times 5 \times 4 \times 7 = (5 \times 4) \times (5 \times 7) = 20 \times 35 = 700$

Number 6 was lovely - I was impressed by this one, but I was intrigued by the fact that students were encouraged to find another way, so that after this, they could discuss which methods were efficient - in this case it was agreed that 1, 2 and 5 were more efficient, certainly than 3 and 4.

This was followed by 125 x 88, for which, the students now used two methods.

1. $(125 \times 8) \times 11 = 1000 \times 11 = 11000$
2. $125 \times (80 + 8) = 125 \times 80 + 125 \times 8 = 10000 + 1000 = 11000$
3. $125 \times (8 + 80) = 125 \times 8 + 125 \times 80 = 1000 + 10000 = 11000$

Interestingly, though 2 & 3 were the same law, it was written on the board, for clarity and completeness. Then to continue the theme of efficiency:

125 x 8888:

1. $(125 \times 8) \times 1111 = 1000 \times 1111 = 1111000$
2. $125 \times (8000 + 800 + 80 + 8) = 1000000 + 100000 + 10000 + 1000 = 1111000$

Here method 1 is far more efficient. I imagine we might think that because these are special cases, we might interpret this as a trick and thus reflect negatively on the usefulness, but it illustrated why students in Shanghai, were then so able to manipulate algebra in middle school. The had this grounding in number (with the algebraic generalisations) that encouraged them to play and explore the structure, so that when challenging algebra was presented, the manipulation was not new, merely the context.

Anyway, this was not the point that I wanted to pass, though I think it is important. Instead, I asked the Chinese teacher if they might have a student that could do 125 x 8888 in their head as they have seen the pattern - he replied yes, but they must show the process.

I pondered how many times our students do not show their workings and how infuriating this is. So I asked, how they know they need to show the process - he replied that in Shanghai, the assessments have two command words.

These are in Chinese, but have a good translation: Show me the answer (just give the solution) or Show me the process (method must be given) - How simple?!

I think this is another thing we could easily adopt and I think this would help us distinguish clearly for the students our expectations. [as opposed to only in questions in exams like "Give Reasons"]

Day 5 – Inverse – if you know one way, you know the inverse…

Hello everyone, an observation today on something that we have seen very consistently this week – that if you learn one concept or principle, then you also know its inverse. This is reinforced throughout Shanghai practice and is echoed in both primary and secondary curriculums.

We use this idea somewhat superficially in the UK, where we might ask students to check answers by the inverse, such as use FOIL to check factorisation, or substitute an answer into the equation from which it was obtained.

However, we do witness the issue of not teaching in this way, where students even at A-level understand that $\frac{x}{6}+\frac{1}{6}=\frac{x+1}{6}$ but when faced with $\frac{x+1}{6}$ do not comfortably see that it can be written as $\frac{x}{6}+\frac{1}{6}$ . This lack of understanding when they first learn fractions leads to further difficulties later, for example at A-Level, partial fractions appears to be a new and difficult concept and not simply an application of everything they already know about algebraic fractions.

In Shanghai, when students learn adding fractions, they will generalise $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$ and the questions, would not simply be:

1. $\frac{1}{6}+\frac{1}{5}$ 2. $\frac{1}{6}+\frac{2}{5}$ 3. $\frac{5}{6}+\frac{1}{5}$ etc…

Instead they might use fill in the blanks to allow students to think in both directions:

2. $\frac{1}{\_}+\frac{1}{5}=\frac{\_}{30}$ 2. $\frac{13}{24}=\frac{\_}{6}+\frac{3}{8}$

In question 2, you could rearrange to find $\frac{13}{24}-\frac{3}{8}$ but it is equally, if not more efficient to do $\frac{13}{24}=\frac{4x+9}{24}$ which means 4x+9=13 and x = 1. Both methods show the use of the inverse and illustrate fluency. In fact, we saw a lesson where the main point of the lesson was to show how you could write unit fractions as the sum of two different unit fractions ($\frac{1}{6}=\frac{1+6}{6(1+6)}=\frac{1+6}{42}=\frac{1}{42}+\frac{6}{42}=\frac{1}{42}+\frac{1}{7}$

There have also been further examples;

Circumference of a circle: C = πd but also $\frac{C}{\pi}=d$

Simplifying and Equivalent fractions: $\frac{a}{b}=\frac{a\times k}{b\times k}=\frac{a\div n}{b\div n}$ so 2 questions given 1/. $\frac{45}{20}$ and 2/. 0.65 : 1.3

Ratio - if a : b = c : d then $\frac{a}{b}=\frac{c}{d}$ and vice versa, leading to two questions 1/. 60 : x = 2 : 5 and 2/. $\frac{x}{20}=\frac{11}{4}$

And in the lesson on the commutative, distributive and associative laws, for example, if we know a(b + c) = ab + ac, then we can answers 1/. 3(x + 2) = 3x + 6 and also 3x + 6 = 3(x + 2).

This is definitely we can try to emphasise with our students, both so that they become more fluent, but more significantly, because by doing this, we teach our students less and they understand more.

Day 6 - A universal curriculum…

As you might imagine there are many cultural differences between the UK and China.

Some of these cultural differences are at the root of the success of the maths curriculum in China. For example, students get maths homework (short – 1-5 questions) every night and at least 90% complete it in a class of over 40 students. A good student will work until 10pm every evening to ensure they have understood all their work – this is not an expectation it is a cultural reality.

I saw an advert for a yoghurt drink on the subway and it had pictures of maths and school books, so the selling point of the yoghurt was doing well at school – this marketing concept would be laughed off the table in the UK.

Thankfully, there is plenty we can adopt and aspire to.

Another big difference is the fact that in Shanghai, there is one textbook – one curriculum – which every school follows. This is the scheme of work in every school which has been carefully crafted by university professors that have time to consider sequencing and structure to the nth degree – plus it is tweaked every couple of years.

This has many benefits: Teachers, instead of all planning a different way to teach the same topic, spend time thinking about ways to ensure students understand the maths content. There is scope to deviate from the examples used and the delivery, but the content and sequencing, which is the fundamentals of mathematical structure remain constant. Thus, if a student moves class, school, area etc… they do not miss anything. All students get a similar experience in every classroom in every school. Schools then use TRG (Teacher research groups) to help ensure that the best interpretation of the maths content is consistently delivered and every teacher learns from one another. If we are honest about the UK, two students in different classes might be taught the same topic in an entirely different way, based on that teacher's experience and subject knowledge, both teachers might be excellent, both lessons might be excellent, but this is unlikely. It is more likely that one lesson might be better than the other (and for another topic it might be the other way around) – ultimately, students do not get the fairest deal and we work too hard. It cannot be efficient for us all to be writing our own lessons for the same content delivery. In Shanghai, the sequence and content is agreed, they have suggested examples to illuminate the structure and then teachers plan from this material – so all they need to do is tweak and interpret. While we cannot do this for the whole country – this is something we can consider and have a go at.

Day 7 – In General…

One of the big differences in practice from the UK to Shanghai, is the constant generalisation that can be seen in practice. They still spend time exploring principles with number, but once a pattern has been recognised, they will generalise with algebra.

One such area of maths where we do this in the UK maths classroom, is the laws of indices: where we might show examples such as x^8/x^3 = x^5 etc… before writing x^m/x^n = x^m-n. In Shanghai, this practice is applied to all aspects of maths.

Significantly, because of the way they use their boards (everything is written up and stays for the whole lesson so it can be referred back to easily and students can use key facts)

I have seen this in nearly every lesson I have observed, for example:

- $\frac{a}{b} = \frac{a \times k}{b \times k} = \frac{a \div n}{b \div n}$ For how fractions can be simplified and equivalents found
- $\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c}$ For adding and subtracting fractions with the same denominator
- $a^2 - b^2 = (a + b)(a – b)$ For completing the sqaure
- $a^2 \pm 2ab + b^2 = (a \pm b)^2$ For squares – this one is particularly useful I think as how many times do our students see $(x – 3)^2$ and write $x^2 - 9$. If our students were fluent, they would know this was the wrong expansion as the result is the difference of two squares.
- Commutativity: (addition) a + b = b + a (multiplication) a x b = b x a
- Associativity: (addition) (a + b) + c = a + (b + c) (multiplication) (a x b) x c = a x (b x c)
- Distributivity: a(b + c) = ab + ac [Can be written a x (b + c) = a x b + a x c]
- Principle of division (this was new to me): $a \div b = \frac{a \div c}{b \div c}$ $c \neq 0$ and used to answer the question $2019 \div 2019\frac{2019}{2020} + \frac{1}{2021}$ which was during a year 8 class!
- Even in geometry lessons, although there was not the same algebraic generalisation, the key principles were written down to be referred to throughout future variations. So in a lesson introducing reflection in a line the following key points were made
    - The object and image points/vertices must be perpendicular
    - The object and image must be congruent
    - The perpendicular distance from the line of reflection of any point on the object and image must be equal

It is lucky for Shanghai teachers that Chinese is much more succinct than English, because these principles seem to be a few characters each, but it is a method we can adopt to support learners more regularly, not only to help them with the topic being studies but expose them to algebra and formal methods more often.

Day 8 – Teacher Research Groups - TRGs

You may have noticed me mention TRGs in previous messages. These Teacher research groups have been the best CPD for subject knowledge and maths pedagogy that I have experienced.

These would be great to emulate the experience for us in the UK.

Every lesson which is observed, is observed by at least 3 members of staff. The member of staff who delivered the lesson will meet with these members of staff on the same day, around a conference table and then there is a strict format:

1. The teacher that designed and delivered the lesson, briefly outlines it - as it happened and why decisions were made about content and questions.
2. Staff who observed will then ask questions about the lesson – specifically following these headings:
    a. Structure – the sequence of the mathematics and the structure revealed – how effective it was at helping students make progress
    b. Content – the questions/examples used and how effective these were at delivering the progress expected by the lesson structure
    c. One other small focus – this might be something the teacher is developing (behaviour/engagement etc…) or something else that warrants attention

These have all been conducted with a culture of lesson improvement. The focus is not on the member of staff, because everyone will have to teach this content and they are working together to explore the best ways to deliver the content. Instead of staff feeling under pressure or scrutinised, they work together with other staff to self-improve and to help develop the teaching of those observing (either they learn a new method, or by improving the lesson they are thinking about their own practice).

We have already made small strides towards adopting a TRG model by having paired observations in the UK, but the next step would be to have periodic open classrooms, where staff could come and observe then meet later to work on the content. If we are going to make the curriculum both effective and impactful, then TRGs could be an essential element in the improvement process.

I plan to run some TRGs in the new year, where I will teach some lessons and invite observers, with a meeting after school to hold a short 20-30 minute TRG. I hope that it will be both enjoyable and inspiring, as I have here in China.

Day 9 – Teaching Techniques

One response to the universal curriculum, is that it could be boring for the students as it is relatively formulaic. First, I would argue that the interest is generated by the variation used and also we know that maths is revered and valued highly here. However, today I saw a teacher who was teaching the common curriculum and content, but that employed a variety of techniques to maintain engagement and focus. In particular, he had lots of ways to get students to explore examples (One thing that aided this was that their interactive whiteboards are 80 inch touchscreen windows based computers so they can link up with staff phones via an app):

1. Teacher led example – teacher writes on the board from own knowledge
2. Teacher led example – teacher writes on the board using student responses
3. Teacher led example – teacher writes each line and asks students to explain the step
4. Student led example – student writes each line on the board and teacher explains
5. Student led example – student writes each line on the board and student explains
6. Student led example – student writes a line, another writes the next
7. Student led example – student writes on the board with one mistake, another corrects
8. Variation – Teacher takes a photo in the lesson and displays it to show method
9. Variation – Teacher takes a photo in the lesson and displays it to show mistake and explain
10. Variation – Teacher takes a photo in the lesson and displays it to show mistake and student corrects
11. Variation – Teacher takes 2 photos in the lesson and displays them to compare methods
12. Variation – Teacher takes many photos over time and uses them to review the topic (see day 3)

I am sure there are more, but we saw many of these in each lesson, and as their lessons are an intense 40 minutes, with short exercises to show knowledge and understanding, to have these different approaches to achieve similar goals, showed that Shanghai teachers also consider pedagogy as well as employing a didactic method (simply presenting information).

Again, it illustrated, that teachers can consider the finer details of their craft, when attitudes to learning do not have to be a consideration. In another example of this, all the teachers plan their board work before a lesson – not just a powerpoint, also the whiteboard work that accompanies it – Which would be great if we had the time, to think to this depth of detail. Nonetheless, these are definitely ideas we can adopt to add variety in our classrooms.

Day 10 – From Shanghai with love …..

My last day in schools and I am full of many emotions – humility, sadness, exhaustion and excitement. I will return to England full of ideas and ready employ everything I have learned from the visit. However, like it is with any training, it is vital that you use it quickly so that it does not become background knowledge.

So, I thought it would be useful to highlight the takeaways so that I have a final summary and some new skills to work on.

I have added a 10th one, which is about board work design.

1. Good Curriculum Design:
    a. Identify the key learning points
    b. Decide and justify the sequence
    c. Identify the structure of the mathematics/key principles (each lesson should have tis focus)
    d. Design questions that reveal this structure to students
2. Number fluency supports algebra fluency and vice-versa.
3. Collect student mistakes to use for a review lesson at the end of each topic.
4. Use the mantra – "Don't just show me the answer, Show me the process!"
5. If you know something one way, you know the inverse – include inverse examples in our teaching.
6. I have a vision of an agreed common curriculum (for each of the 3 sets)
7. Use generalisation to stress the key principles that students should apply
8. TRGs are the best subject knowledge and pedagogy CPD experiences I have had
9. Different methods of displaying worked examples to engage students
10. Board work design – leave worked examples up for the entire lesson where possible.

I could also add a lot on Variation, but we will explore this concept at a later stage!