National Centre for Excellence in the Teaching of Mathematics

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NCETM Mathematics Textbook Guidance

This paper is written primarily for authors and publishers of mathematics textbooks for school-age pupils (that is, from Key Stage 1 to Key Stage 5). The NCETM hopes that this paper will also be of use to teachers (this term includes parents / carers) who are comparing available textbooks prior to purchase.

The paper sets out the philosophy, structure and features that the NCETM expects to see in a mathematics textbook. The NCETM does not endorse specific publishers or authors, and this paper should not be seen as an exhaustive "check-list".

Summary

*A high quality mathematics textbook is an educational resource that can be used by pupils both **in lessons and independently**, and that also provides both **subject knowledge and pedagogy support** to teachers of mathematics. It is a comprehensive learning tool, providing support for the development of both **procedural fluency** and **conceptual understanding in mathematics**: a textbook consisting only of worked examples and simple repetitive exercises is unlikely to develop these. It is essential that the principles under-pinning teaching with **variation**[1] are reflected in the choice of examples and the structure of the exercises. Pupils must be provided with frequent opportunities for **intelligent practice.**[2]*

The NCETM's principles of a well-designed and well-written textbook are set out in greater detail below. These have been drawn from international research; academic references can be found at the end of this document.

Mathematical coherence

Mathematical coherence, both within and across textbook material, is essential in order for mathematical ideas to be connected, so that pupils achieve depth of understanding and make the logical connections necessary in order to progress. To achieve this, it is necessary that:

- The mathematical content of the textbook, or the series of textbooks, is presented in a coherent and connected manner, exemplifying a logical, step by step approach across the age range(s) it addresses.

- The learning sequence is set out in carefully organised sections with clear lines of progression, showing how current ideas develop from previous work and are interconnected.

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[1] *A reference for understanding variation theory is the paper by Gu L., Huang, R., & Marton, F. (2004) "Teaching with variation: A Chinese way of promoting effective mathematics learning" from "How Chinese learn mathematics: Perspectives from insiders" (p. 309).*

[2] *Intelligent practice is a term used to describe practice exercises that integrate the development of fluency with the deepening of conceptual understanding. Attention is drawn to the mathematical structures and relationships to assist in the deepening of conceptual understanding, whilst at the same time developing fluency through practice.*

- Concepts are presented and developed in a detailed, step by step, logical approach, to avoid gaps in learning.

- The text is accessible to the age of the pupils and is clear and concise in the language it uses.

- Correct and precise mathematical vocabulary is introduced early, and pupils' subsequent use and understanding of formal mathematical language is developed in a coherent and consistent manner.

- When new concepts are introduced, these are related to key concepts with which the pupils are already familiar, and explicit connections are made to prior learning.

- There is a balance of opportunities to develop procedural fluency and conceptual understanding and the materials integrate these two aspects of learning.

- Where appropriate, the textbook materials provide a clear idea of the relevance of the mathematical ideas and how these are used to solve problems. However, the textbook does not downplay or undermine the truth that mathematics is the study of abstract ideas: spurious claims of relevance are not made.

- Related concepts are frequently presented together, for example pairs of inverse operations such as addition and subtraction, multiplication and division or exponentiation and taking logarithms, in order to expose and explore important mathematical relationships and to develop pupils' fluency.

Mistakes, misconceptions and misunderstandings

"Getting things wrong" is a vital part of the natural process of learning. Technical / procedural errors (mistakes), errors of application or identification (misunderstandings) and errors which have defensible logical foundations (misconceptions) all give pupils highly valuable opportunities to develop their mathematical skills. Learning is deepened as much by learners recognising what's incorrect as what's correct. It is necessary, therefore that:

- There is explicit reference to and use of mistakes, misconceptions and misunderstandings, for example through exercises where pupils are asked to identify correct and incorrect procedures, reasoning or solutions.

- Aspects of the mathematical ideas that are likely not to be immediately clear to pupils are highlighted and explored in depth.

- The textbook takes account of classic, standard or predictable mistakes, misconceptions and misunderstandings associated with the topic under consideration, and it provides information that enables teachers to anticipate and plan how to tackle these.

- Examples of concepts are presented alongside “non-examples”, for example asking pupils to distinguish between shapes that are and aren’t polygons, or functions that are or aren’t invertible, or variables with a linear relationship that are or aren’t directly proportional to each other. These provide valuable opportunities for pupils to contrast and compare.

Mathematical tasks and exercises

Mathematical tasks and exercises support progression in learning through engaging pupils in mathematical thought. Worthwhile tasks and exercises deepen conceptual understanding and embed procedural fluency. To achieve this, it is necessary that:

- Careful thought is given to the nature, structure and sequence of tasks and exercises to expose the underlying structure of concepts and the mathematical relationships between the current and prior learning.

- Tasks provide the appropriate level of challenge and do not oversimplify the mathematics. Unjustified ‘Quick Tips’ to achieve superficial success are avoided.

- Not all tasks and exercises lead to a ‘closed form’ numerical or algebraic answer. Some tasks and exercises require pupils to comment on answers and results, or to say what they have noticed about a set of answers and thence make conjectures, generalisations and reasoned mathematical arguments.

- Some tasks and questions include the use of engaging and appropriate contexts relevant both to the age of the pupil and also to the mathematics being explored.

- Tasks are varied and involve pupils in a range of cognitive processes including consolidation and practice, investigation, conjecturing and hypothesising, explaining, reasoning, proving, applying, interpreting and analysing.

Representing mathematics

Learning mathematics requires pupils to engage in abstract thinking; the ultimate goal is for pupils to be able to identify with justification and apply, with fluency, flexibility and accuracy, mathematical models, techniques and theories to explore ideas, deepen understanding and solve problems. However in order to achieve this level of sophistication, mathematics needs to be represented in ways that provide pupils with access to and understanding of mathematical concepts. It is necessary, therefore, that:

- The textbook uses clear and appropriate representations of mathematics that provide insight and understanding of the concepts being taught. For example the bar model, used in Singapore and other high performing countries, is a powerful model applicable in both primary and secondary mathematics lessons. The bar representation exposes the mathematical structure of

the problem being considered, enabling the pupil to see with clarity the concepts and procedures needed to solve the problem.

- Representations of the mathematics in the form of pictures and diagrams are used to provide access to the mathematics, revealing the underlying structures and helping pupils make sense of mathematical ideas.

- Illustrations and contexts used are relevant and appropriate to the particular mathematical ideas and concepts, and are not used merely for decoration or attractive presentation.

Use of technology

Careful consideration should be given as to how and when technology is used to support learning in mathematics, to ensure it does not detract from the development of essential knowledge and skills. This is likely to be achieved when:

- Any supporting online materials are linked to the textbook and mirror its structure and coherence, and are not a set of random downloads.

- Any accompanying digital software is used appropriately and enhances rather than detracts from the focus of the learning. For example it might exploit the opportunity for dynamic interaction to reason about and explore mathematical concepts or provide opportunity for practice in mental calculation.

- Calculators as digital tools are not used to replace the development of essential mental and written calculation skills.

Outside the classroom

Mathematical modelling, reasoning, predicting and testing are key life skills; their development should not be restricted to the mathematics classroom. This is achieved when:

- The textbook provides focused homework and out of school activities that give the opportunity for practice and application, and ensure the development of both conceptual understanding and procedural fluency.

- Appropriate and meaningful opportunities are sometimes provided, to encourage pupils to think about and apply mathematics to everyday contexts and other curricular subjects.

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Assessment and review

Assessment is an integral part of effective teaching and learning, and textbook materials should provide good assessment support. This is achieved when:

- Both formative and summative assessments of learning are incorporated, allowing both pupils and teachers to measure progress and inform future learning.
- Assessment activities regularly allow pupils the opportunity to review the key ideas and concepts and to check their own understanding. They are not merely a collection of more questions to practise.
- Assessment of depth of understanding and fluency to support deep sustainable learning is incorporated.

Educative teacher material

The textbook should be educative for teachers as well as pupils. It should support teachers' ongoing development of their subject knowledge and their pedagogical practice. This is achieved when:

- Publishers of textbooks are committed to ensuring the necessary teacher professional development to support effective use of the materials is available.
- A comprehensive teacher's guide accompanies the textbook materials. This provides support for teacher subject knowledge enhancement and mathematics specific pedagogy, as well as guidance on effective use of the textbook and supporting materials. It also provides insight into why the tasks and exercises have been selected and the key points that should be drawn out.

Textbook authors

Research shows that textbooks are highly influential on teachers' pedagogy, the development of subject knowledge, and pupil attainment. Therefore:

- Only authors with expert knowledge, skills and credibility should be selected.
- Stringent and independent quality assurance measures must be applied to the authors' work.
- Authors must draw on research evidence and accrued professional experience.
- Materials must be trialled and refined in the light of trial data, wherever it is possible to do so.

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